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RADC-TR-68-19

SPACING PERTURBATION TECHNIQUES FOR ARRAY OPTIMIZATION

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TECHNICAL REPORT NO. RADC-TR-68-19
November 1967

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Monitored by: Rome Air Development Center
under Contract No. F30602-68-C-0067

Rome Air Development Center
Air Force Systems Command
Griffiss Air Force Base, New York

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FOREWORD

This report was prepared by Messrs F. I. Tseng and David K. Cheng of the Electrical Engineering Department, College of Engineering, Syracuse University Research Institute, Syracuse, New York under RADC Contract No. F30602-68-C-0067, ARPA Order No. 1010.

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by RADC Project Engineer Joseph Lovecchio (EMATS).

SPACING PERTURBATION TECHNIQUES FOR ARRAY OPTIMIZATION

by

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ABSTRACT - A spacing perturbation technique is developed for the maximization of the directive gain and the signal-to-noise ratio of a linear array of identical elements. The initial array can be of uniform or nonuniform spacings with any given excitation amplitude and phase distribution. After an optimum set of spacings is obtained by perturbation, the excitation amplitudes and phases can be adjusted for further improvement in the performance index of interest. The optimum spacings can be recalculated and the cycle of iteration repeated if desired. The spacing perturbation technique is based on a new optimization theorem; there is no need to solve nonlinear equations or to rely on a trial-and-error procedure. In all cases computed, the iteration process converges rapidly. Illustrative examples are given for both broadside and endfire operations. Typical radiation patterns are plotted which bring out a number of interesting features of the optimization process.

I. INTRODUCTION

The optimization of appropriate performance indices of discrete antenna arrays is a problem of primary importance for antenna engineers. The directive gain and the output signal-to-noise ratio are two of the more important performance indices. A number of articles (Uzkov, 1946; Uzsoky and Solymar, 1956; Tai, 1964) have dealt with the problem of gain maximization for linear arrays with uniformly spaced elements. Methods are also available (Cheng and Tseng, 1965; Lo, Lee and Lee (1966), for the maximization of the directive gain of antenna arrays with an arbitrary configuration. When the performance index of interest is the signal-to-noise power ratio at the output of an array, it is also possible to determine the optimum amplifications and phase shifts in the array elements (Cheng and Tseng, 1966). Tseng and Cheng (1967a,b) further extended their studies on gain and signal-to-noise ratio maximization to include random variations in element positions and in excitation amplitudes and phases.

All of the above-mentioned articles started with an array of a given configuration, and optimization is achieved by properly adjusting the excitation amplitudes and phases in the array elements. However, for a given set of amplitude and phase values, uniform spacing does not yield the highest obtainable gain or signal-to-noise ratio. Recently Butler and Unz (1967) tackled the problem of determining the current distribution and the element spacings which will combine to give a best possible beam efficiency or gain, resulting in arrays with nonuniform

spacings. They found the optimum element positions by checking the spacing deviations in the direction of the maximum change in the largest eigenvalue of a pencil of matrices. An approximate, perturbation method was also discussed. It appears that their method is quite tedious to apply and its success depends critically upon a proper initial choice of element spacings which must be close to the optimum. The signal-to-noise performance of the array was not considered.

This paper presents a spacing perturbation technique for the optimization of the directive gain or the signal-to-noise ratio of an array. The starting point can be an array of arbitrary (uniform or nonuniform) spacings with nonoptimum excitation amplitudes and phases. The basis of the technique lies in an optimization theorem which will be proved in the Appendix. After an optimum set of spacings is obtained, the excitation amplitudes and phase shifts can be adjusted for further improvement in the desired performance index; then the optimum spacings can be recalculated for the new excitation and the cycle repeated if desired. In particular, the technique provides a method for improving the gain or the output signal-to-noise ratio of an array with any given excitation and spacings by spacing perturbation until a maximum is obtained. Illustrative examples for both broadside and endfire arrays are included and some interesting numerical results are given.

II. GAIN OR SNR AS THE PERFORMANCE INDEX

We consider a linear array of $2N + \eta$ identical antenna elements symmetrically located about the origin, with $\eta = 1$ when the total number of elements is odd, and $\eta = 0$ when the total number of elements is even and

the center element is absent. Let θ_o be the angle which the direction of the signal makes with the array axis. The antenna elements do not have to be isotropic, but they will be assumed to be symmetrically excited. Hence, if the excitation in the mth element from the origin is $I_m \exp(j\Phi_m)$, that in the -mth element on the other side is $I_m \exp(-j\Phi_m)$, where

$$\Phi_m = -(2\pi d_m/\lambda) \cos\theta_o - \phi_m \quad . \tag{1}$$

In (1), d_m is the distance of the mth element from the origin, λ is the signal wavelength, and ϕ_m is the phase shift from the cophasal operation. The array factor will then be

$$E(\psi) = \eta \frac{I_o}{2} + \sum_{m=1}^{N} I_m \cos(D_m\psi - \phi_m) \quad , \tag{2}$$

where

$$\psi = (2\pi d/\lambda)(\cos\theta - \cos\theta_o) \tag{3}$$

$$D_m = d_m/d \tag{4}$$

and d, a normalizing distance, may be any choice of convenience. For example, if one starts with a uniformly spaced array, it would be natural to make d the spacing between neighboring elements. We define the output signal-to-noise ratio as the ratio of the power received per unit solid angle in the direction of the signal to the average noise power received per unit solid angle. Thus,

$$G_{SNR} = \frac{|E(0)|^2}{\frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} |E(\psi)|^2 \, w(\theta, \phi) \sin\theta \, d\theta} \quad , \tag{5}$$

where

$$w(\theta, \phi) = g(\theta, \phi)\, T(\theta, \phi) \tag{6}$$

$$g(\theta, \phi) = \text{element power pattern function} \tag{7}$$

$$T(\theta, \phi) = \text{spatial distribution function of noise power} \tag{8}$$

It is convenient to normalize $g(\theta, \phi)$ in (7) with respect to its value in the direction of the signal; i.e., $g(\theta_o, \phi_o) = 1$. The composite function $w(\theta, \phi)$ in (6) can be viewed as a weighting function on the array power pattern $|E(\psi)|^2$. We note that (5) becomes the expression for directive gain, G_o, when $T(\theta,\phi) = 1$; hence it serves as the starting point for the optimization of both G_{SNR} and G_o. Clearly, both are affected by the normalized element positions $\{D_m\}$ and the excitations $\{I_m, \phi_m\}$. We shall consider their effects separately in the following sections.

III. OPTIMIZATION BY SPACING PERTURBATION

Let an array of $2N + \eta$ elements be given by specifying its element power pattern function $g(\theta, \phi)$, its normalized element positions $\{D_m^o\}$ and its element excitations $\{I_m \exp(j\Phi_m)\}$. We wish to determine the required spacing perturbations such that G_{SNR} in a specified direction will be maximized in a given noise environment $T(\theta, \phi)$. Let the perturbed normalized element positions be $\{D_m\}$.

$$D_m = D_m^o + x_m , \tag{9}$$

where x_m represents the spacing perturbation for the mth element and $x_m \ll 1$. Substitution of (9) in (2) yields approximately

$$E(\psi) = E^{o}(\psi) - \sum_{m=1}^{N} x_m [I_m \psi \sin (D_m^{o}\psi - \phi_m)], \tag{10}$$

where $E^{o}(\psi)$ is the unperturbed array factor with D_m^{o} substituted for D_m in (2). Using (10), we can write (5) in the following form:

$$G_{SNR} = \frac{|E^{o}(0)|^2}{A - 2\bar{x}'\bar{\beta} + \bar{x}'\bar{\bar{C}}\bar{x}}, \tag{11}$$

where

$$A = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} |E^{o}(\psi)|^2 \, w(\theta, \phi) \sin\theta \, d\theta \tag{12}$$

$$\bar{x}' = [x_1, x_2, \ldots x_m, \ldots, x_N] \tag{13}$$

is the transpose of the column matrix of spacing perturbations $\bar{x}$; $\bar{\beta}$ is a column matrix of typical element

$$\beta_m = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} I_m \psi E^{o}(\psi) \, w(\theta, \phi) \sin (D_m^{o}\psi - \phi_m) \sin\theta \, d\theta \; ; \tag{14}$$

and $\bar{\bar{C}} = [c_{mk}]$ is an $N \times N$ square matrix with

$$c_{mk} = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} I_m I_k \psi^2 w(\theta,\phi) \sin (D_m^{o}\psi - \phi_m) \sin (D_k^{o}\psi - \phi_k) \sin\theta \, d\theta. \tag{15}$$

It can readily be shown that $\bar{\bar{C}}$ is symmetric and positive definite. Use can then be made of the theorem proved in the Appendix, which enables us to conclude:

a) $$\underset{(\bar{x} = \bar{x}_M)}{\text{Max. } G_{SNR}} = \frac{|E^o(0)|^2}{A - \bar{\beta}' \bar{\bar{C}}^{-1} \bar{\beta}} \tag{16}$$

b) $$\bar{x}_M = \bar{\bar{C}}^{-1} \bar{\beta} . \tag{17}$$

Equations (16) and (17) give the results of a first-order perturbation. After the components of $\bar{x}_M$ have been determined from (17), they can be substituted back in (9). One can then use $(\bar{D}^o + \bar{x}_M)$ as the new normalized element-position column matrix and perform a second-order perturbation to obtain further improvement in the performance index. This process can be repeated until it becomes evident that further iteration yields a negligible improvement. For the many cases we have computed, some of which will be presented in sections V and VI as illustrative examples, it is found that convergence toward optimum values usually takes place very quickly; seldom are more than two iterations required.

IV. OPTIMIZATION BY EXCITATION ADJUSTMENTS

The spacing perturbation technique developed in the preceding section yields the required element positions in order to maximize G_{SNR} for a given set of excitation parameters. Even if one starts with a uniformly spaced array, the element spacings will no longer be uniform after the perturbation. Now this perturbed, nonuniformly spaced array can be further optimized by proper amplifications and phase shifts following the array elements. With this in mind, we let $y_o = I_o$, $y_n = I_n \cos \phi_n$, and $y_{N+n} = I_n \sin \phi_n$ (n=1,2,...,N). Furthermore, let $h_o(\psi) = \eta/2$, $h_n(\psi) = \cos D_n\psi$, and $h_{N+n}(\psi) = \sin D_n\psi$ (n=1,2,...,N). Equation (5) can then be converted to the following form:

$$G_{SNR} = \frac{\bar{y}' \bar{h}_o \bar{h}_o' \bar{y}}{\bar{y}' \bar{\bar{B}} \bar{y}} , \tag{18}$$

where

$$\bar{y}' = [y_o, y_1, y_2, \ldots, y_{2N}] \tag{19}$$

$$\bar{h}_o' = [h_o(0), h_1(0), h_2(0), \ldots, h_{2N}(0)] \tag{20}$$

and $\bar{\bar{B}} = [b_{ij}]$ is a $(2N + \eta) \times (2N + \eta)$ square matrix with

$$b_{ij} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} h_i(\psi)\, h_j(\psi)\, w(\theta, \phi) \sin\theta \, d\theta \, . \tag{21}$$

G_{SNR} in (18) is expressed as a ratio of two quadratic forms. Since $\bar{\bar{B}}$ is symmetric and positive definite, we know (Cheng and Tseng, 1965) that G_{SNR} can be maximized by choosing the column matrix $\bar{y} = \bar{y}_M$:

$$\bar{y}_M = \bar{\bar{B}}^{-1} \bar{h}_o \tag{22}$$

and that

$$\underset{(\bar{y} = \bar{y}_M)}{\text{Max. } G_{SNR}} = \bar{h}_o' \, \bar{\bar{B}}^{-1} \, \bar{h}_o \, . \tag{23}$$

Equation (22) completely specifies the amplifications and phase shifts required to obtain the maximum possible G_{SNR}, as given in (23), for the perturbed array. If the performance index of interest is directive gain G_o, $\bar{y}_M$ will yield the required excitation amplitudes and phases to maximize G_o. We have now reached a second submaximum, which may possibly be further improved by holding the excitation unchanged and again perturbing the spacings. The cycle may be repeated until further adjustments are no longer worthwhile.

We emphasize that this alternate spacing perturbation and excitation adjustment procedure of seeking a maximum performance index can be applied to

an array which initially has an arbitrary, nonuniform spacing and an arbitrary distribution of excitation amplitudes and phases. One could start this double optimization procedure either by perturbing the element spacings first or by adjusting the excitation amplitudes and phases first. Some typical examples of both approaches will be presented in the following two sections.

V. EXAMPLES FOR GAIN OPTIMIZATION

We consider a symmetrical linear array of seven isotropic elements. For isotropic elements $g(\theta, \phi) = 1$ and the expressions in (14), (15) and (21) for β_m, c_{mk} and b_{ij} respectively are simplified. We shall examine the broadside and endfire operations separately. In each case the initial interelement spacings are chosen to be uniform and of such a value as to yield a maximum gain with a uniform cophasal excitation.

(a) Broadside Operation ($\theta_o = 90^o$)

The characteristics of the initial array are: $D_m^o = d_m^o/d = m$, $d = 1.77(\lambda/2)$, $I_m^o = 1$, $\Phi_m^o = 0^o$, for all m. This uniformly excited, equally spaced, 7-element broadside array has a gain of 11.32. By keeping the amplitude uniform and perturbing the interelement spacings, the gain can be increased to 11.51. Although this is not a big increase, we note that it equals the value achieved by adjusting the amplitudes and keeping the spacings uniform (Tai, 1964). If we further optimize our space-perturbed array by amplitude adjustments, using the method discussed in section IV, we obtain the results listed in the third row of Table I. The final data for the doubly optimized array by repeating the above spacing perturbation and amplitude adjustment cycle are shown as the last

row of Table I. The optimized array has a gain of 11.63 which is only a few percent higher than that for the unperturbed uniform array. This implies that an equally spaced and uniformly excited broadside array at $d = 1.77(\lambda/2)$ is very close to the optimum arrangement. The excitation amplitudes and element positions of one-half of the symmetrical optimized array are depicted in Fig. 1(a). The radiation patterns for both the equally spaced, uniformly excited array and the optimized array are shown in Fig. 1(b). The patterns do not differ much; but it is reassuring to find that the optimized array also has a better sidelobe structure.

(b) Endfire Operation ($\theta_o = 0^\circ$)

For endfire operation a uniformly excited, equally spaced array has a gain maximum when the interelement spacing is around 0.84 $\lambda/2$. The corresponding results obtained by (1) spacing perturbation, (2) excitation adjustments after spacing perturbation, and (3) iteration of the above two steps are tabulated in Table II. In this case optimization by spacing perturbation and excitation adjustments is more effective in increasing the gain. The final optimized, nonuniformly excited and unequally spaced array is seen to have a gain of 20.0, an increase of 85 percent from the original value of 10.8.

VI. EXAMPLES FOR SNR OPTIMIZATION

Calculations for the optimization of G_{SNR} depends, of course, on the spatial distribution function of noise power. For illustration purpose, we assume that the noise distribution has no ϕ-variation and has the form of $T(\psi)$, as shown in Fig. 2. It is consisted essentially of two parts: one a

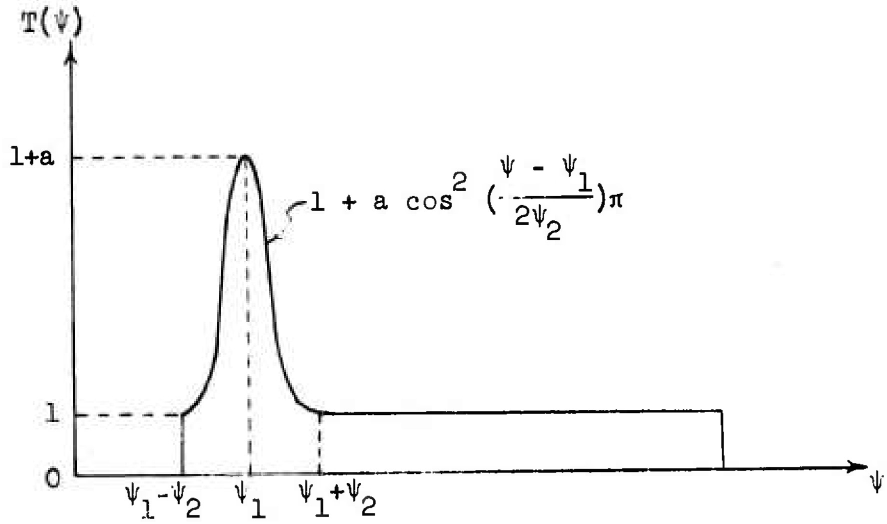

Fig. 2 - Spatial Distribution of Noise Power.
$\psi = (2\pi d/\lambda)(\cos\theta - \cos\theta_o)$.

constant part and the other a superimposed part which is assumed to be of a cosine-squared form and of a finite extent. The superimposed part can be used to simulate an interference or clutter distribution whose amplitude, location and extent can be varied by changing $\underline{a}$, ψ_1, and ψ_2 respectively. The termination point of the $T(\psi)$ curve is left unspecified because it depends on whether the array under consideration is operating in a broadside or an endfire mode. A large quantity of numerical data have been obtained. Some typical results follow.

(a) <u>Broadside Operation ($\theta_o = 90^o$)</u>

Again we consider a linear array of seven isotropic elements. The parameters chosen for $T(\psi)$ in Fig. 2 are: $\underline{a} = 15$, $\psi_1 = (1/4)(2\pi d/\lambda)$, $\psi_2 = (1/12)(2\pi d/\lambda)$. The results for the (1) space-perturbed, (2) excitation-adjusted (by amplification), and (3) optimized arrays are listed in Table III. We note that large improvements in G_{SNR} are possible by optimization through

either spacing perturbation or excitation adjustments and that the G_{SNR} of the optimized array is about 11.4 times that of the original uniform array. Of course, the array optimized for a maximum SNR is not the same as the one for a maximum gain. This can be seen readily by comparing the excitation amplitudes and interelement spacings in Tables I and III. The directive gain of the SNR-optimized array in Table III (sketched in Fig. 3(a)) is 10.6. The radiation patterns are plotted in Fig. 3(b), in which the spatial distribution of the noise or interference power is also shown. It is interesting to see that, at the expense of a slightly wider main beam, the sidelobes of the optimized array are everywhere lower than those of the uniform array. In particular, the first sidelobe, which normally occurs in a region where the noise power is high, is much suppressed and its position slightly shifted.

(b) <u>Endfire Operation ($\theta_o = 0^o$)</u>

For computation in the 7-element endfire case, we use a $T(\psi)$ similar to the broadside case above; but ψ_1 and ψ_2 values are chosen differently on account of a different visible range as well as a different sidelobe structure. The results are tabulated in Table IV and plotted in Fig. 4. We see several noteworthy points from this example. First, G_{SNR} is increased from a value of 15.8 for the uniform, cophasal and equally spaced array to 891.4 (56.4 times). The absolute G_{SNR} values for the broadside and endfire cases (e.g.: 16.0 versus 15.8, and 181.9 versus 891.4) are not too meaningful because the noise power distribution function $T(\psi)$ has not been normalized with respect to the different $|E^o(0)|^2$ values. However, the percentage of possible improvement in G_{SNR} is significant in each case, and we see that one

can do much more for endfire operation. Secondly, Fig. 4 shows that the improvement in G_{SNR} is accompanied by a better sidelobe structure with an emphatic suppression in the region of high noise power. Thirdly, computation reveals that, along with the vast increase in G_{SNR}, the directive gain of the SNR-optimized array is also increased to 15.5 from 10.4 for the uniform cophasal array.

VII. CONCLUSION

Based upon a new optimization theorem, a spacing perturbation technique has been presented for the maximization of the directive gain and the signal-to-noise ratio of an array of identical elements. The technique can be applied to an array which initially has nonuniformly spaced elements with arbitrary excitation amplitudes and phases. After an optimum set of spacings is obtained by perturbation, the excitation amplitudes and phase shifts can be adjusted for further improvement in the performance index of interest. The optimum spacings can be recalculated and the cycle of iteration repeated if desired. In all cases computed, the iteration process converges very rapidly, seldom requiring more than two cycles. The present technique does not require the solution of nonlinear equations, and it does not involve a trial-and-error procedure. Numerical examples and typical radiation patterns are included. It is noted that the final, optimized array will always have nonuniform inter-element spacings and is therefore relatively insensitive to frequency variations.

VIII. APPENDIX

Theorem. If a quantity P can be expressed in terms of an $N \times 1$ real column vector $\bar{x}$ as

$$P = A - 2\bar{x}'\bar{\beta} + \bar{x}' \bar{\bar{C}} \bar{x} , \tag{24}$$

where A is a constant, $\bar{\beta}$ is another $N \times 1$ real column vector, $\bar{x}'$ is the transpose of $\bar{x}$, and $\bar{\bar{C}}$ is an $N \times N$ positive definite, symmetric, square matrix, then

a)
$$\min_{(\bar{x} = \bar{x}_M)} P = A - \bar{\beta}' \bar{\bar{C}}^{-1} \bar{\beta} \quad , \quad \text{and} \tag{25}$$

b)
$$\bar{x}_M = \bar{\bar{C}}^{-1} \bar{\beta} \tag{26}$$

Proof: If $\bar{\bar{C}}$ is positive definite, it is known that (Gantmacher, 1959)

$$(\bar{\beta}' \bar{\bar{C}}^{-1} \bar{\beta})(\bar{x}' \bar{\bar{C}} \bar{x}) \geq (\bar{x}' \bar{\beta})^2 \tag{27}$$

or

$$\bar{x}' \bar{\bar{C}} \bar{x} \geq \frac{1}{\bar{\beta}' \bar{\bar{C}}^{-1} \bar{\beta}} (\bar{x}' \bar{\beta})^2 \quad , \tag{28}$$

where the equality sign applies when

$$\bar{x} = \bar{x}_M = \bar{\bar{C}}^{-1}\bar{\beta} \quad . \tag{29}$$

Let $c = \bar{\beta}' \bar{\bar{C}}^{-1} \bar{\beta} > 0$, and $b = \bar{x}' \bar{\beta}$. We have, from (24) and (28),

$$\begin{aligned} P &= A - 2b + \bar{x}' \bar{\bar{C}} \bar{x} \\ &\geq A - 2b + \frac{b^2}{c} \end{aligned} \tag{30}$$

But,

$$\begin{aligned} A - 2b + \frac{b^2}{c} &= A - c + \frac{1}{c} (c-b)^2 \\ &\geq A - c \end{aligned} \tag{31}$$

Combining (30) and (31), we obtain

$$P \geq A - \bar{\beta}' \, \bar{\bar{C}}^{-1} \, \bar{\beta} \,, \tag{32}$$

where the equality sign holds with (29); hence the theorem is proved.

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TABLE I

Gain Optimization for Broadside Array–Initial Array:
7 isotropic elements, equally spaced,
uniformly excited, and cophasal. ($d_o = 0$)

	I_o	I_1	I_2	I_3	$(d_1-d_o)\frac{2}{\lambda}$	$(d_2-d_1)\frac{2}{\lambda}$	$(d_3-d_2)\frac{2}{\lambda}$	G_o
Uniform array	1.00	1.00	1.00	1.00	1.77	1.77	1.77	11.32
Spaced-perturbed array	same as above				1.81	1.78	1.65	11.51
Exc.-adjusted array	1.00	1.00	0.98	0.85	same as above			11.61
Optimized array	1.00	0.99	0.96	0.82	1.79	1.76	1.66	11.63

TABLE II

Gain Optimization for Endfire Array–Initial array:
7 isotropic elements, equally spaced,
uniformly excited, and cophasal. ($d_o = 0$)

	Excitations				$(d_1-d_o)\frac{2}{\lambda}$	$(d_2-d_1)\frac{2}{\lambda}$	$(d_3-d_2)\frac{2}{\lambda}$	G_o
Uniform cophasal array	I_o=1.00 ϕ_o=0°	I_1=1.00 ϕ_1=0°	I_2=1.00 ϕ_2=0°	I_3=1.00 ϕ_3=0°	0.84	0.84	0.84	10.8
Space-perturbed array	same as above				0.91	0.90	0.82	11.5
Exc.-adjusted perturbed array	I_o=1.00 ϕ_o=0°	I_1=1.00 ϕ_1=7.9°	I_2=0.99 ϕ_2=17.2°	I_3=0.89 ϕ_3=37.2°	same as above			14.6
Optimized array	I_o=1.00 ϕ_o=0°	I_1=0.99 ϕ_1=16.6°	I_3=0.96 ϕ_2=35.9°	I_3=0.73 ϕ_3=77.7°	0.86	0.84	0.71	20.0

TABLE III

SNR Optimization for Seven-Element Broadside Array

$$a = 15, \quad \psi_1 = \frac{1}{4}\left(\frac{2\pi d}{\lambda}\right), \quad \psi_2 = \frac{1}{12}\left(\frac{2\pi d}{\lambda}\right), \quad d_o = 0.$$

	I_o	I_1	I_2	I_3	$(d_1-d_o)\frac{2}{\lambda}$	$(d_2-d_1)\frac{2}{\lambda}$	$(d_3-d_2)\frac{2}{\lambda}$	G_{SNR}
Uniform array	1.00	1.00	1.00	1.00	1.77	1.77	1.77	16.0
Space-perturbed array	same as above				1.65	1.76	2.10	19.8
Exc.-adjusted array	1.00	0.89	0.67	0.39	same as above			78.1
Optimized array	1.00	0.86	0.59	0.40	1.66	1.72	1.74	181.9

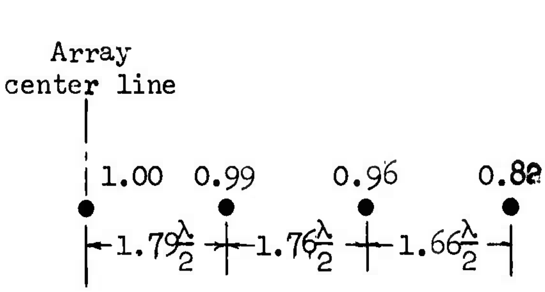
TABLE IV

SNR Optimization for Seven-Element Endfire Array

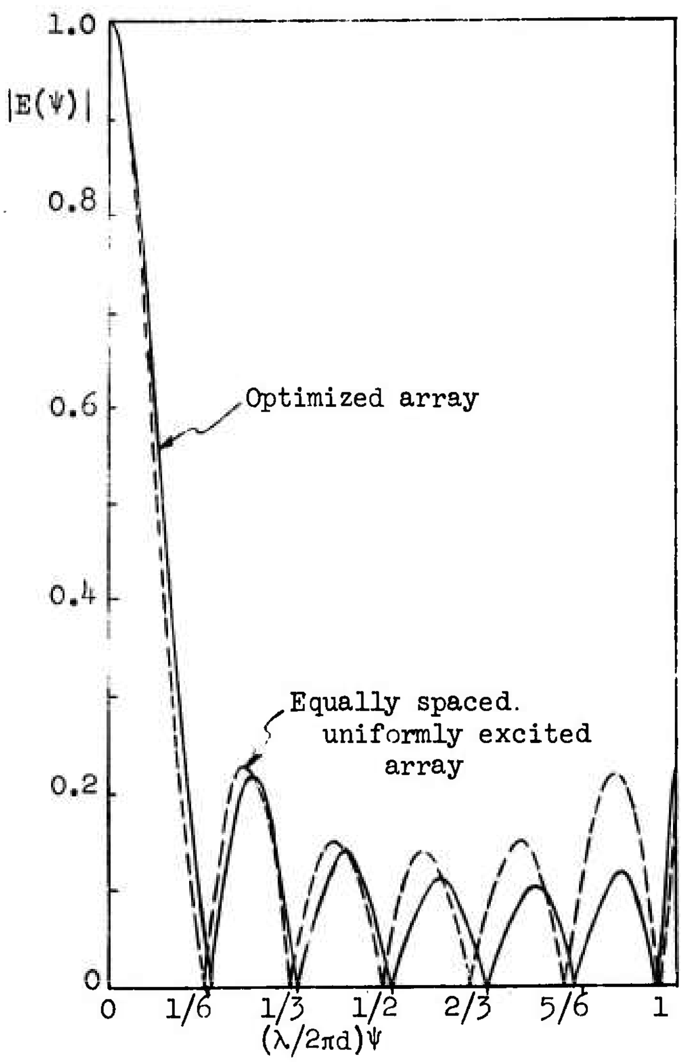
$$a = 15, \quad \psi_1 = \frac{1}{2}\left(\frac{2\pi d}{\lambda}\right), \quad \psi_2 = \frac{3}{20}\left(\frac{2\pi d}{\lambda}\right), \quad d_o = 0.$$

$$\Phi_m = - 0.84\, m\pi - \phi_m$$

	Excitations				$(d_1-d_o)\frac{2}{\lambda}$	$(d_2-d_1)\frac{2}{\lambda}$	$(d_3-d_2)\frac{2}{\lambda}$	G_{SNR}
Uniform cophasal array	I_o=1.00	I_1=1.00	I_2=1.00	I_3=1.00	0.84	0.84	0.84	15.8
	ϕ_o=0°	ϕ_1=0°	ϕ_2=0°	ϕ_3=0°				
Space-perturbed array	same as above				0.65	0.80	0.96	26.6
Exc.-adjusted perturbed array	I_o=1.00	I_1=0.75	I_2=0.34	I_3=0.14	same as above			745.7
	ϕ_o=0°	ϕ_1=43.5°	ϕ_2=57.7°	ϕ_3=40.2°				
Optimized array	I_o=1.00	I_1=0.75	I_2=0.34	I_3=0.14	0.64	0.78	0.93	891.4
	ϕ_o=0°	ϕ_1=45.4°	ϕ_2=61.8°	ϕ_3=50.6°				

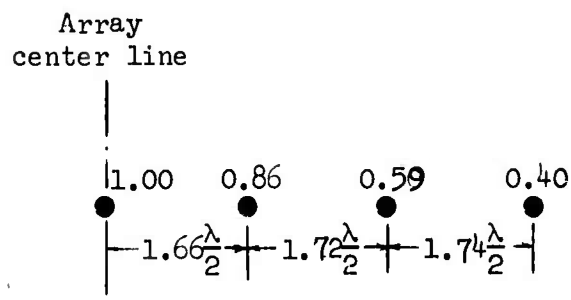


(a) Gain optimized array

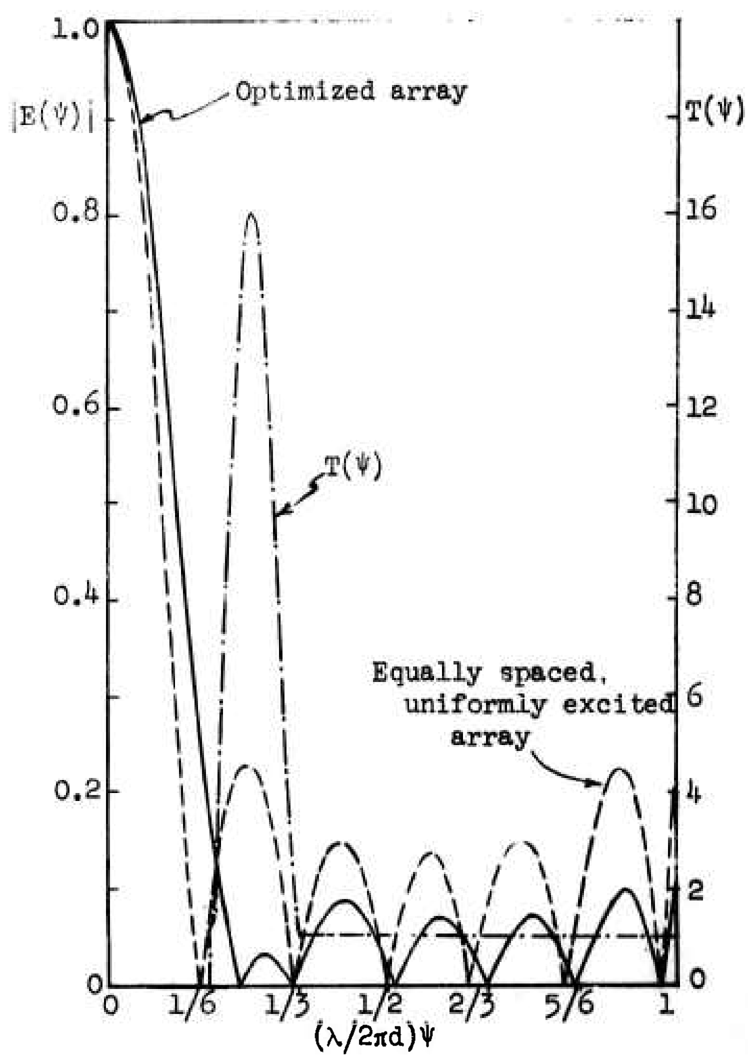


(b) Radiation patterns

Fig. 1. Gain optimization for seven-element broadside array.



(a) SNR optimized array



(b) Radiation patterns

Fig. 3. SNR optimization for seven-element broadside array.

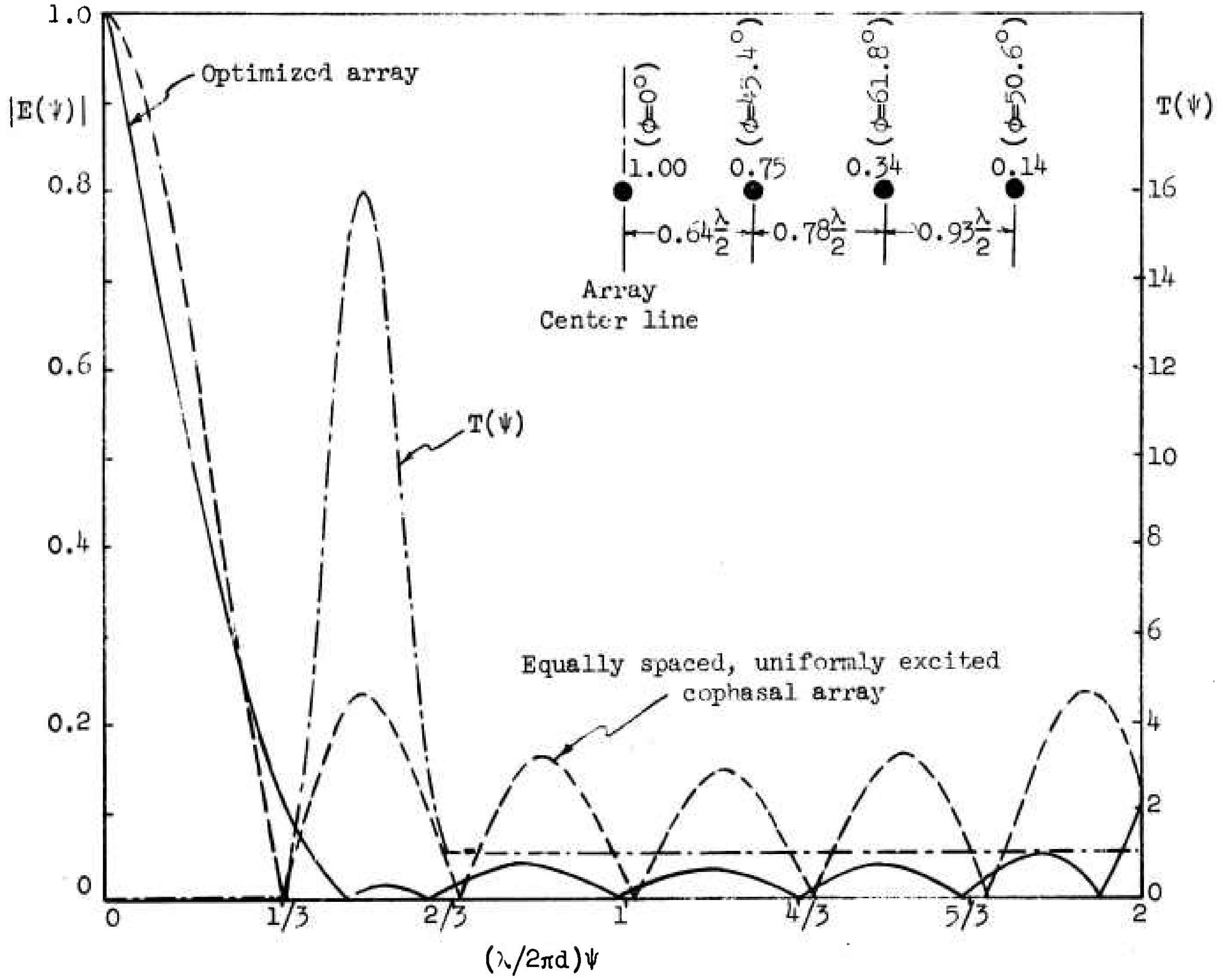


Fig. 4. SNR optimization for seven-element endfire array.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Syracuse University Research Institute
Syracuse, New York

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

Spacing Perturbation Techniques for Array Optimization

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Interim

5. AUTHOR(S) (First name, middle initial, last name)

F. I. Tseng
David K. Cheng

6. REPORT DATE

January 1968

7a. TOTAL NO. OF PAGES

7b. NO. OF REFS

8a. CONTRACT OR GRANT NO.

F30602-68-C-0067

b. PROJECT NO.

ARPA Order No. 1010

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

Technical Report #1

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

RADC-TR-68-19

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11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

RADC (EMATS)
Griffiss AFB, New York 13440

13. ABSTRACT

A spacing perturbation technique is developed for the maximization of the directive gain and the signal-to-noise ratio of a linear array of identical elements. The initial array can be of uniform or nonuniform spacings with any given excitation amplitude and phase distribution. After an optimum set of spacings is obtained by perturbation, the excitation amplitudes and phases can be adjusted for further improvement in the performance index of interest. The optimum spacings can be recalculated and the cycle of iteration repeated if desired. The spacing perturbation technique is based on a new optimization theorem; there is no need to solve nonlinear equations or to rely on a trial-and-error procedure. In all cases computed, the iteration process converges rapidly. Illustrative examples are given for both broadside and endfire operations. Typical radiation patterns are plotted which bring out a number of interesting features of the optimization process.

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Antenna Array Theory						
Antenna Array Optimization						
Antenna Array Spacing						